5.1 Exercise Set

1. Fill in the blanks.

a) A function of the form $y = f(x) = ax^2 + bx + c$ is called a quadratic function.

b) A function of the form $y = f(x) = ax^2 + bx + c$ has a graph called a parabola.

c) The lowest point on a graph that opens upward is the min. of the parabola.

d) The vertex is the highest point on a parabola that opens down.

e) The line that divides a parabola into two separate halves is called axis of symmetry.

f) A parabola is a polynomial of degree 2.

g) A parabola $y = f(x) = ax^2 + bx + c$ with $a > 0$, is a parabola that opens up.

h) A parabola $y = f(x) = ax^2 + bx + c$ with $a < 0$, is a parabola that opens down.

i) The point on a parabola $y = f(x) = ax^2 + bx + c$ where $f(0) = c$, is the y-int of the parabola.

j) The point on a parabola $y = f(x) = ax^2 + bx + c$ where $f(x) = 0$, is the x-int of the parabola.

2. Which functions are quadratic functions?

a) $y = x^2 + 4$ **b)** $y = 3x - 2$ (linear) **c)** $y = x^2 + \sqrt{2}x$ **d)** $y = x^2 + 2\sqrt{x}$ **e)** $y = x^2 + \frac{x}{3}$ **f)** $y = x^2 + \frac{3}{x}$

3. Find the x-intercept(s) (if possible), y-intercept , and axis of symmetry of the following quadratic functions.

a) $y = f(x) = x^2 - 6x + 9$ $y = (x-3)^2$
x-int: (3,0) aos: $x = 3$
y-int: 9 (0,9)

b) $y = g(x) = x^2 - x - 2$

c) $y = h(x) = 2x^2 + x - 6$ $(2x-3)(x+2)$
x-int: $3/2, -2$. aos: $\frac{1.5-2}{2}$ $x = -\frac{1}{4}$
y-int: -6

d) $y = j(x) = 9 - x^2$

e) $y = k(x) = -x^2 + 2x + 3$ $-(x^2-2x-3) \to -(x-3)(x+1)$
x-int: 3, -1 aos: $x = 1$
y-int: 3

f) $y = l(x) = x^2 + 1$

g) $y = m(x) = -x^2 - 2x + 8$ $-(x^2+2x-8) \to -(x+4)(x-2)$
x-int: $-4, 2$. aos: -1
y-int: 8

h) $y = n(x) = 4 - 4x + x^2$

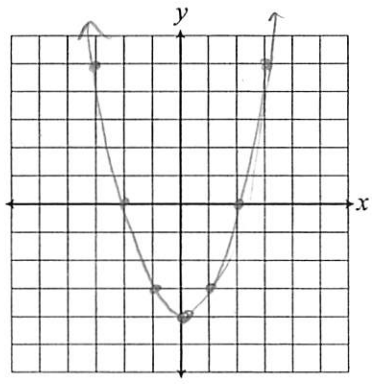
i) $y = p(x) = x^2 - \sqrt{2}x$ $x(x-\sqrt{2})$
x-int: $0, \sqrt{2}$
y-int: 0 aos: $\frac{\sqrt{2}}{2} = x$.

j) $y = q(x) = -\sqrt{6}x^2 - \sqrt{2}x$

4. Graph the given quadratic function by completing a table of values.

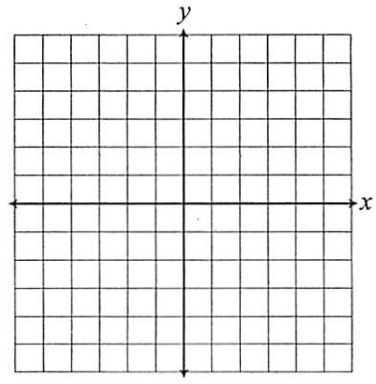
a) $f(x) = x^2 - 4$

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5



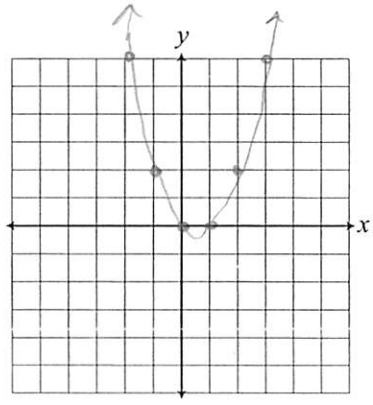
b) $g(x) = -x^2 + 4$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



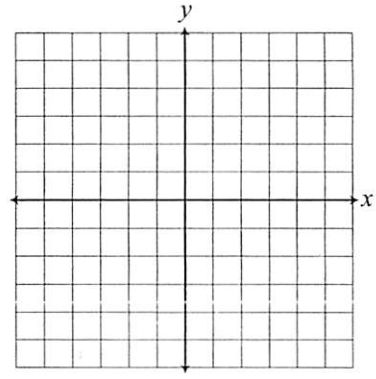
c) $h(x) = x^2 - x$

x	y
-3	12
-2	6
-1	2
0	0
1	0
2	2
3	6



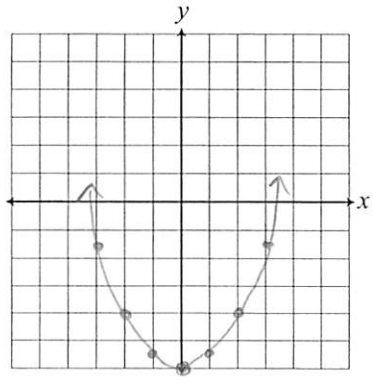
d) $j(x) = -x^2 + x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



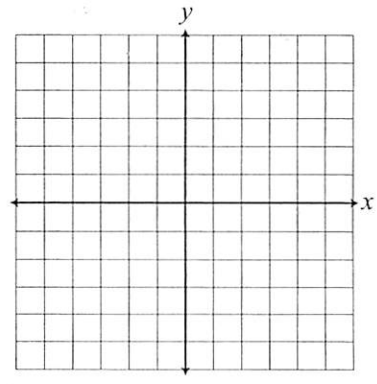
e) $k(x) = \frac{1}{2}x^2 - 6$

x	y
-3	-1.5
-2	-4
-1	-5.5
0	-6
1	-5.5
2	-4
3	-1.5



f) $l(x) = -2x^2 + 6$

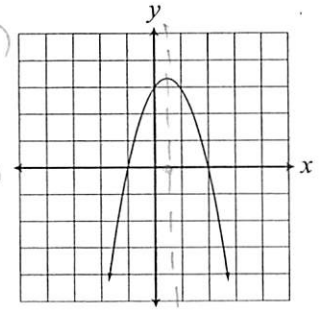
x	y
-3	
-2	
-1	
0	
1	
2	
3	



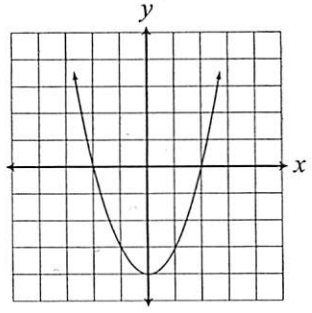
5. Find the following features of the given parabolas.

i) Vertex
ii) Axis of symmetry
iii) x-intercept(s)
iv) y-intercept
v) Domain of f
vi) Range of f
vii) Maximum or minimum value

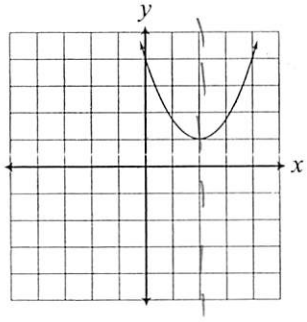
a) *i*) (0.5, 3.3)
ii) $x = 1/2$
iii) (−1, 0)(2, 0)
iv) (0, 3)
v) $x \in R$
vi) $y \leq 3.3$
vii) max 3.3.



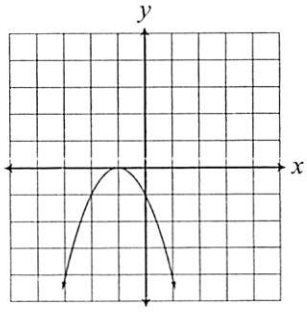
b) *i*) _____
ii) _____
iii) _____
iv) _____
v) _____
vi) _____
vii) _____



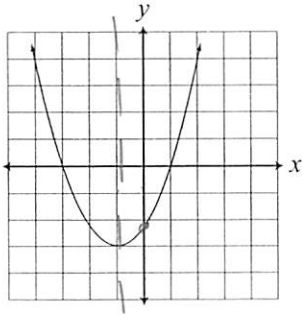
c) *i*) (2, 1)
ii) $x = 2$
iii) n/a
iv) (0, 4)
v) $x \in R$
vi) $y \geq 1$..
vii) min 1



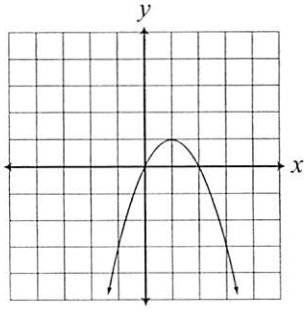
d) *i*) _____
ii) _____
iii) _____
iv) _____
v) _____
vi) _____
vii) _____



e) *i*) (−1, −3)
ii) $x = -1$
iii) (−3, 0)(1, 0)
iv) (0, −2.2)
v) $x \in R$
vi) $y \geq -3$
vii) min −3



f) *i*) _____
ii) _____
iii) _____
iv) _____
v) _____
vi) _____
vii) _____



6. Find a third point on the quadratic function that has the indicated vertex, and whose graph passes through the given point.

a) vertex $(3,2)$, point $(5,7)$ over 2 up 5 $(1,7)$

b) vertex $(-1,4)$, point $(2,5)$ _____

c) vertex $(3,0)$, point $(7,4)$ over 4 up 4 $(-1,4)$

d) vertex $(2,-5)$, point $(-3,2)$ _____

e) vertex $(-4,2)$, point $(-7,8)$ over 3 $(-1,2)$

f) vertex $(-1,-4)$, point $(3,-6)$ _____

g) vertex (c,d), point $(-3,1)$ $(2c+3,1)$

$c+x=-3$

$x=-3-c$

$c-(-3-c)=2c+3$

h) vertex $(-3,1)$, point (c,d) _____

i) vertex (a,b), point (c,d) $(2a+c,d)$

j) vertex (c,d), point (a,b) _____

7. If the graph of a quadratic function passes through the given points, find the equation of the axis of symmetry. Does the parabola open upward or downward?

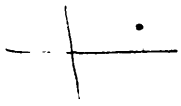
a) $(0,0),(5,0),(3,1)$

$x=2.5$

opens down

b) $(-6,3),(5,-2),(-2,3)$

c) $(5,2),(2,8),(6,8)$

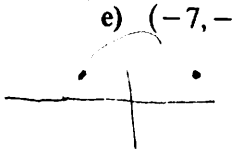


$x=4$

opens up.

d) $(-4,-1),(-8,-3),(0,-1)$

e) $(-7,-5),(-3,2),(4,2)$



$x=0.5$

opens down.

f) $(-3,4),(6,6),(5,4)$

g) $(a,b),\left(\frac{a}{2},c\right),(-a,b),\ b>c$

$x=0$

opens up

h) $(a,b),\left(\frac{a}{2},c\right),(-3a,b),\ b<c$

5.2 Exercise Set

Match the equation of the quadratic function with the corresponding graph.

a) $f(x) = 2(x-1)^2 - 3$ ii

b) $f(x) = -2(x+1)^2 - 3$ vii

c) $f(x) = -2(x-1)^2 + 3$ v

d) $f(x) = -2(x-1)^2 - 3$ viii

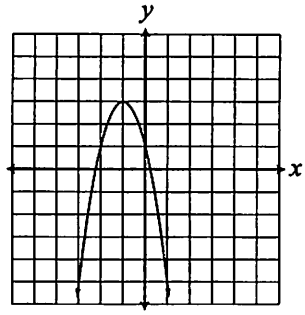
e) $f(x) = 2(x-1)^2 + 3$ iv

f) $f(x) = -2(x+1)^2 + 3$ i

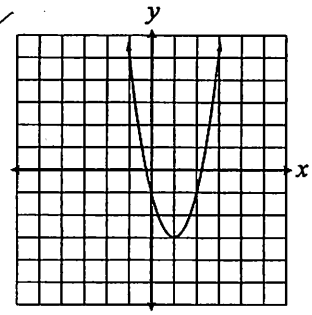
g) $f(x) = 2(x+1)^2 - 3$ vi

h) $f(x) = 2(x+1)^2 + 3$ iii

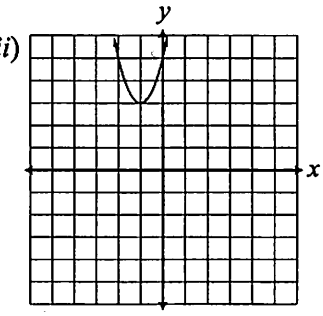
i)



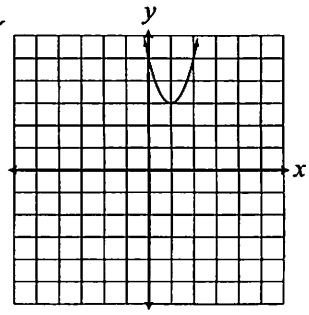
ii)



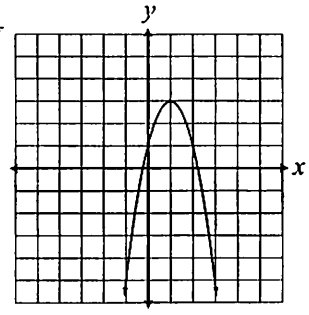
iii)



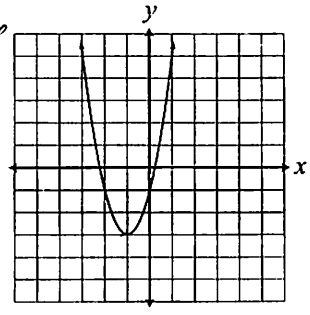
iv)



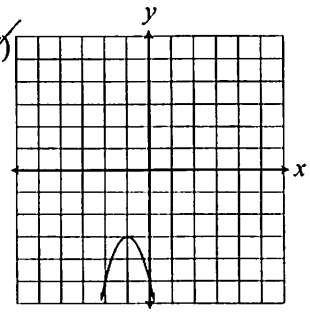
v)



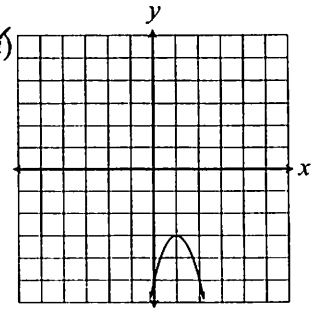
vi)



vii)



viii)



2. Graph each function $y = f(x) = a(x-h)^2 + k$, using at least 5 points. Determine the:

i) Vertex
ii) Axis of symmetry
iii) y-intercept
iv) x-intercepts
v) Domain of f
vi) Range of f
vii) Maximum / minimum value

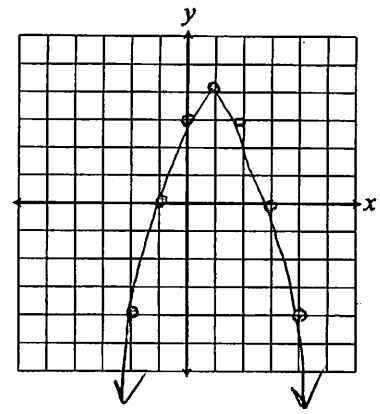
a) $y = -(x-1)^2 + 4$

i) (1,4) *ii)* $x=1$

iii) (0,3) *iv)* (−1,0) (3,0)

v) $\mathbb{R}$ *vi)* $y \leq 4$

vii) max of 4



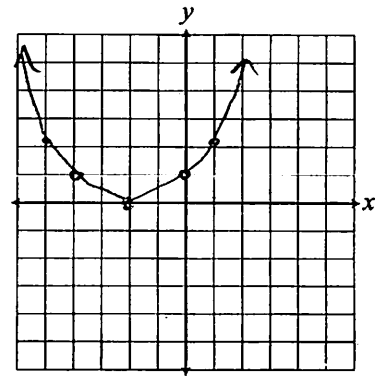
b) $y = \frac{1}{4}(x+2)^2$

i) (−2,0) *ii)* $x=-2$

iii) (0,1) *iv)* (−2,0)

v) $\mathbb{R}$ *vi)* $y \geq 0$

vii) min of 0



c) $y = -\frac{1}{2}x^2 + 6$

$0 = -\frac{1}{2}x^2 + 6$

$-6 = -\frac{1}{2}x^2$

$12 = x^2$

$\pm\sqrt{12} = x$

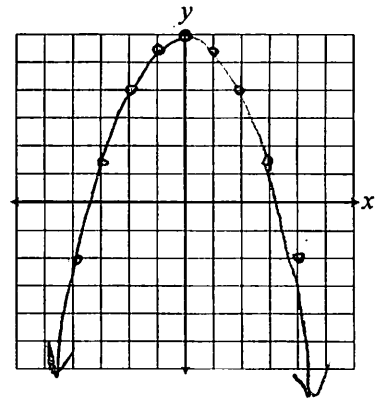
$\pm 2\sqrt{3}$

i) (0,6) *ii)* $x=0$

iii) (0,6) *iv)* $(\pm 2\sqrt{3}, 0)$

v) $\mathbb{R}$ *vi)* $y \leq 6$

vii) max of 6



$0 = -2(x+1)^2 + 3$

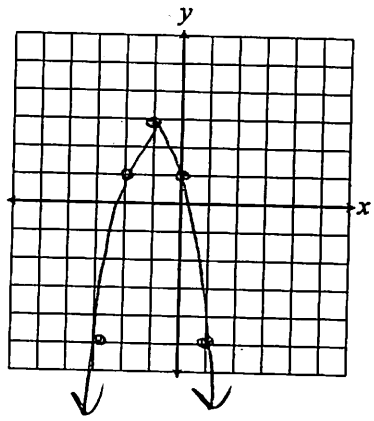
$-3 = -2(x+1)^2$

$\frac{3}{2} = (x+1)^2$

$\pm\sqrt{\frac{3}{2}} = x+1$

$-1 \pm \sqrt{\frac{3}{2}} = x$

25. **d)** $y = -2(x+1)^2 + 3$



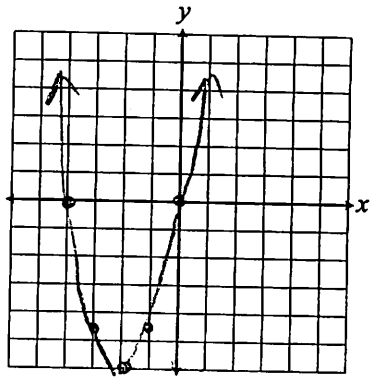
i) $(-1, 3)$ *ii)* $x = -1$

iii) $(0, 1)$ *iv)* $\left(-1 \pm \sqrt{\frac{3}{2}}, 0\right)$

v) $\mathbb{R}$ *vi)* $y \leq 3$

vii) max 3

e) $y = \frac{3}{2}(x+2)^2 - 6$



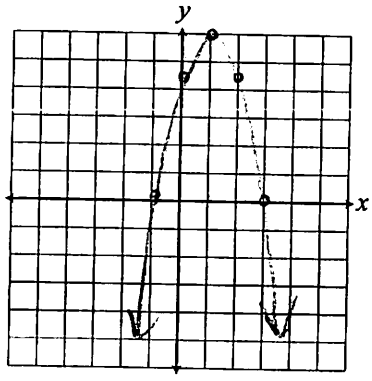
i) $(-2, -6)$ *ii)* $x = -2$

iii) $(0, 0)$ *iv)* $(0, 0)(-4, 0)$

v) $\mathbb{R}$ *vi)* $y \geq -6$

vii) min -6

f) $y = -\frac{3}{2}(x-1)^2 + 6$



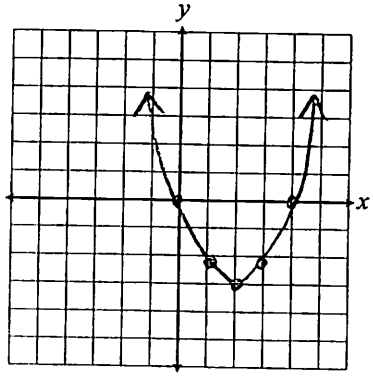
i) $(1, 6)$ *ii)* $x = 1$

iii) $(0, 4.5)$ *iv)* $(-1, 0)(3, 0)$

v) $\mathbb{R}$ *vi)* $y \leq 6$

vii) max 6

g) $y = \frac{3}{4}(x-2)^2 - 3$



i) $(2, -3)$ *ii)* $x = 2$

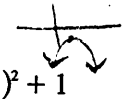
iii) $(0, 0)$ *iv)* $(0, 0)(4, 0)$

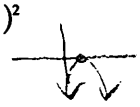
v) $\mathbb{R}$ *vi)* $y \geq -3$

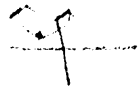
vii) min -3

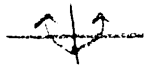
3. Find the number of x-intercepts for the following quadratic functions.

a) $f(x) = -2(x-1)^2 + 1$  2

b) $g(x) = -2(x-1)^2 - 1$  0

c) $h(x) = -2(x-1)^2$  1

d) $j(x) = 2(x+1)^2 + 1$  0

e) $k(x) = 3x^2 - 1$  2

f) $l(x) = -3x^2 - 1$  0

4. Write an equation for a quadratic function with the same shape as $f(x) = \frac{2}{3}x^2$, but with the given point as a vertex.

a) $(0,2)$ $y = \frac{2}{3}x^2 + 2$

b) $(2,0)$ $y = \frac{2}{3}(x-2)^2$

c) $(-1,3)$ $y = \frac{2}{3}(x+1)^2 + 3$

d) $(1,-3)$ $y = \frac{2}{3}(x-1)^2 - 3$

e) $(4,2)$ $y = \frac{2}{3}(x-4)^2 + 2$

f) $(-2,-4)$ $y = \frac{2}{3}(x+2)^2 - 4$

g) $(-a,-b)$ $y = \frac{2}{3}(x+a)^2 - b$

h) $(a-b,c)$ $y = \frac{2}{3}(x-a+b)^2 + c$

5. Write an equation of a parabola that has the shape $f(x) = \pm 3x^2$, and the given vertex.

a) maximum $(2,0)$
$y = -3(x-2)^2$

b) minimum $(0,-1)$
$y = 3x^2 - 1$

c) maximum $(-1,4)$
$y = -3(x+1)^2 + 4$

d) minimum $(1,-2)$
$y = 3(x-1)^2 - 2$

e) maximum $(\sqrt{2},-\sqrt{3})$
$y = -3(x-\sqrt{2})^2 - \sqrt{3}$

f) minimum $(-1+\sqrt{2},\sqrt{5})$
$y = 3(x+1-\sqrt{2})^2 + \sqrt{5}$

6. Write the equation of a quadratic function $h(x)$ that has the same shape as $f(x) = -\frac{1}{2}(x+1)^2 - 3$ and the same vertex as $g(x) = 2(x+2)^2 + 1$.

$y = -\frac{1}{2}(x+2)^2 + 1$

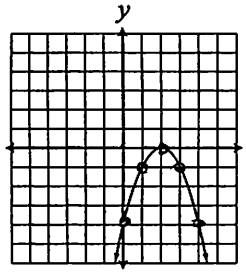
7. Write the equation of a quadratic function $h(x)$ that has the same shape as $f(x) = \frac{2}{3}(x-1)^2 + 2$ and the same vertex as $g(x) = -(x+3)^2 - 4$.

$y = \frac{2}{3}(x+3)^2 - 4$

5.3 Exercise Set

Determine the equation for the parabola.

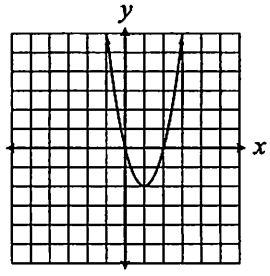
a)



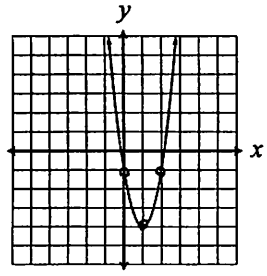
$a=-1$

$y=-(x-2)^2$

b)



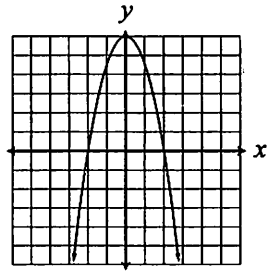
c)



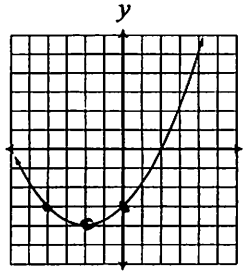
$a=3$

$y=3(x-1)^2-4$

d)



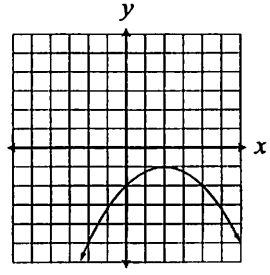
e)



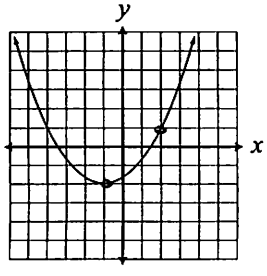
$a=\frac{1}{4}$

$y=\frac{1}{4}(x+2)^2-4$

f)



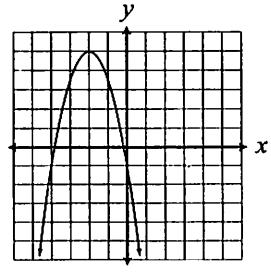
g)



$a=\frac{1}{3}$

$y=\frac{1}{3}(x+1)^2-2$

h)



2. Find the equation of a quadratic function whose graph satisfies the given conditions.

a) vertex: $(2,9)$ x-intercept: 5 $(5,0)$

$y=a(x-2)^2+9$

$0=a(5-2)^2+9$

$-9=a(3)^2$

$-1=a$ $\quad y=-(x-2)^2+9$

b) vertex: $(-2,12)$ x-intercept: -4

c) vertex: $(1,-4)$ x-intercept: -2 $(-2,0)$

$y=a(x-1)^2-4$

$0=a(-2-1)^2-4$

$4=a(-3)^2$

$4/9=a$ $\quad y=\frac{4}{9}(x-1)^2-4$

d) vertex: $(-4,12)$ x-intercept: 4

e) vertex $(-3,-5)$ y-intercept: 1 $(0,1)$

$y=a(x+3)^2-5$

$1=a(0+3)^2-5$

$6=a(3)^2$

$\frac{6}{9}=a$ $\quad y=\frac{2}{3}(x+3)^2-5$

f) vertex: $(2,4)$ y-intercept: -3

g) vertex: $(1,4)$ point: $(2,3)$

$y=a(x-1)^2+4$

$3=a(2-1)^2+4$

$-1=a$ $\quad y=-(x-1)^2+4$

h) vertex: $(-2,-4)$ point $(-3,-1)$

i) vertex: $(0,2)$ point: $\left(\frac{1}{2},\frac{3}{2}\right)$

$y=ax^2+2$

$\frac{3}{2}=a\left(\frac{1}{2}\right)^2+2$ $\quad y=-2x^2+2$

$-\frac{1}{2}=a\left(\frac{1}{4}\right)$ $\quad a=-2$

j) vertex: $(-3,0)$ point: $\left(-\frac{3}{2},\frac{1}{2}\right)$

k) vertex: $(\sqrt{2},5)$ point: $(-3\sqrt{2},-3)$

$y=a(x-\sqrt{2})^2+5$

$-3=a(-3\sqrt{2}-\sqrt{2})^2+5$

$-8=a(-4\sqrt{2})^2$

$-8=a(32)$ $\quad y=-\frac{1}{4}(x-\sqrt{2})^2+5$

$-\frac{1}{4}=a$

l) vertex: $(-\sqrt{3},-6)$ point: $(3\sqrt{3},2)$

3. Find the equation of a quadratic function whose graph contains the given points.

a) $(2,0),(-4,0),(-1,-2)$

$y=a(x+1)^2+q$

$-2=a(-1+1)^2+q$

$-2=q$

$0=a(2+1)^2-2$

$2=a(3)^2 \quad a=\frac{2}{9}$

$y=\frac{2}{9}(x+1)^2-2$

b) $(-3,0),(1,-8),(5,0)$

c) $(-1,0),(0,3),(3,0)$

d) $(-1,0),(0,5),(5,0)$

e) $(-1,0),\left(0,\frac{5}{2}\right),(5,0)$ $\quad x=2$

$\frac{5}{2}=a(0-2)^2+q$

$\frac{5}{2}=4a+q \quad q=\frac{5}{2}-4a$

$0=a(5-2)^2+\frac{5}{2}-4a$

$0=9a+\frac{5}{2}-4a \quad -\frac{5}{2}=8a \quad a=-\frac{1}{2} \quad q=\frac{5}{2}-4\left(-\frac{1}{2}\right)$

$y=-\frac{1}{2}(x-2)^2+\frac{9}{2}$

f) $(-5,0),(-1,0),\left(0,\frac{5}{2}\right)$

g) $(-2,3),(0,3),(-4,-5)$

h) $(-3,-1),(1,-1),(-2,5)$

i) $(-2,1),(-6,1),(2,-7)$ $\quad x=-4$

$-7=a(2+4)^2+q$

$-7=36a+q \quad q=-7-36a$

$1=a(-2+4)^2-7-36a$

$1=4a-7-36a$

$32a=-8 \quad a=-1/4 \quad q=-7-36\left(-\frac{1}{4}\right)=2$

$y=-\frac{1}{4}(x+4)^2+2$

j) $(-4,8),(-3,23),(-8,8)$

4. If a parabola has x-intercepts a and b, what is the x-coordinate of the vertex?

$\frac{a+b}{2}$ (the average)

5. Find all values of k so that the vertex of the graph $y = x^2 + kx + 16$ lies on the x-axis.

6. Determine the quadratic function f, with zeros -2 and 6, and range $y \leq 4$.

(−2, 0) x 4

- a is negative
- aos: $\frac{-2+6}{2} = 2$

$y = a(x-2)^2 + 4$

$0 = a(-2-2)^2 + 4$

$-4 = a(16)$ $a = -1/4$

$y = -\frac{1}{4}(x-2)^2 + 4$

7. Determine the quadratic function f with points $(-3, 5)$ and $(1, 5)$, and range $y \geq -2$.

8. If the graph of the quadratic function $f(x) = ax^2 + bx + c$ passes through the origin, what is the value of c?

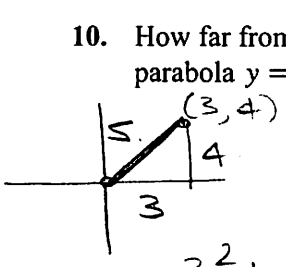
(0,0)

$0 = a(0) + b(0) + c$

$0 = c$.

9. Consider the quadratic equation $y = ax^2 + ax + b$. Find the vertex if $a = 4b$.

10. How far from the origin is the vertex of the parabola $y = -2(x-3)^2 + 4$?



V(3,4)

$3^2 + 4^2 = d^2$

$25 = d^2$

$5 = d$

11. Find the distance between the vertices of the parabolas $y = \frac{1}{2}(x+2)^2 + 6$ and $y = -2(x-4)^2 - 2$.

5.4 Exercise Set

1. Find a number that makes the equation valid.

a) $(x+1)^2 = x^2 + 2x + \underline{1}$

b) $(x-1)^2 = x^2 - 2x + \underline{1}$

c) $(x+2)^2 = x^2 + 4x + \underline{4}$

d) $(x-2)^2 = x^2 - 4x + \underline{4}$

e) $(x+3)^2 = x^2 + 6x + \underline{9}$

f) $(x-3)^2 = x^2 - 6x + \underline{9}$

g) $(x+4)^2 = x^2 + 8x + \underline{16}$

h) $(x-4)^2 = x^2 - 8x + \underline{16}$

2. Find a number that makes the expression a perfect square of the form $(x+h)^2$. divide by 2 → square

a) $x^2 + 2x + \underline{1}$ → 1

b) $x^2 - 2x + \underline{1}$ → −1

c) $x^2 + 4x + \underline{4}$

d) $x^2 - 4x + \underline{4}$ → −2

e) $x^2 + 5x + \underline{\frac{25}{4}}$ → $\frac{5}{2}$

f) $x^2 - 7x + \underline{\frac{49}{4}}$ → $-\frac{7}{2}$

g) $x^2 + bx + \underline{\frac{b^2}{4}}$ → $\frac{b}{2}$

h) $x^2 - bx + \underline{\frac{b^2}{4}}$ → $-\frac{b}{2}$

3. Find a number to make each side equal in value.

a) $x^2 + 6x + \underline{9} = (x + \underline{3})^2$ → 3

b) $x^2 - 10x + \underline{25} = (x - \underline{5})^2$ → −5

c) $x^2 + 5x + \underline{\frac{25}{4}} = (x + \underline{\frac{5}{2}})^2$ → $\frac{5}{2}$

d) $x^2 - 7x + \underline{\frac{49}{4}} = (x - \underline{\frac{7}{2}})^2$ → $-\frac{7}{2}$

e) $x^2 + \frac{2}{3}x + \underline{\frac{1}{9}} = (x + \underline{\frac{1}{3}})^2$ → $\frac{1}{3}$

f) $x^2 - \frac{3}{4}x + \underline{\frac{9}{64}} = (x - \underline{\frac{3}{8}})^2$ → $-\frac{3}{8}$

g) $x^2 + bx + \underline{\frac{b^2}{4}} = (x + \underline{\frac{b}{2}})^2$ → $\frac{b}{2}$

h) $x^2 - \frac{b}{a}x + \underline{\frac{b^2}{4a^2}} = (x - \underline{\frac{b}{2a}})^2$ → $-\frac{b}{2a}$

4. Find the vertex.

a) $f(x) = x^2 + 4x + 3$

$f(x) = (x^2 + 4x + 4 - 4) + 3$

$f(x) = (x+2)^2 - 4 + 3$

$f(x) = (x+2)^2 - 1$ $\quad V(-2, -1)$

b) $g(x) = x^2 + 6x + 10$

c) $h(x) = x^2 - 8x + 15$

$h(x) = (x^2 - 8x + 16 - 16) + 15$

$h(x) = (x-4)^2 - 16 + 15$

$h(x) = (x-4)^2 - 1$ $\quad V(4, -1)$

d) $j(x) = x^2 - 10x + 18$

e) $k(x) = x^2 + 3x - 8$

$k(x) = (x^2 + 3x + \frac{9}{4} - \frac{9}{4}) - 8$

$k(x) = (x + \frac{3}{2})^2 - \frac{41}{4}$ $\quad V(-\frac{3}{2}, -\frac{41}{4})$

f) $l(x) = -x^2 - 3x + 5$

g) $m(x) = 3x^2 - 18x + 25$

$m(x) = 3(x^2 - 6x + 9 - 9) + 25$

$m(x) = 3(x-3)^2 - 27 + 25$

$m(x) = 3(x-3)^2 - 2$ $\quad V(3, -2)$

h) $n(x) = -3x^2 + 5x - 3$

i) $p(x) = \frac{1}{2}x^2 - 3x + 4$

$p(x) = \frac{1}{2}(x^2 - 6x + 9 - 9) + 4$

$p(x) = \frac{1}{2}(x-3)^2 - \frac{9}{2} + 4$ $\quad V(3, -1/2)$

$p(x) = \frac{1}{2}(x-3)^2 - \frac{1}{2}$

j) $q(x) = -\frac{2}{3}x^2 + 4x - 1$

k) $r(x) = 0.6x^2 + 2x - 3$

$r(x) = 0.6(x^2 + \frac{10}{3}x + \frac{25}{9} - \frac{25}{9}) - 3$

$r(x) = 0.6(x + \frac{5}{3})^2 - \frac{5}{3} - 3$ $\quad V(-\frac{5}{3}, -\frac{14}{3})$

$r(x) = 0.6(x + \frac{5}{3})^2 - 14/3$

l) $s(x) = -\frac{3}{4}x^2 + \frac{2}{3}x + \frac{1}{2}$

5. Find the x and y-intercepts, if possible, for each quadratic function.

a) $f(x) = (x-3)^2 - 4$

y-int → $y = (-3)^2 - 4 = 5$

x-int → $0 = (x-3)^2 - 4$

$4 = (x-3)^2$

$\pm 2 = x - 3$.

$x = 3 \pm 2$ $x = 5, 1$

b) $g(x) = (x-3)^2 + 4$

c) $h(x) = 4(x+1)^2 - 36$

y-int → $y = 4(1)^2 - 36 = -32$

x-int → $0 = 4(x+1)^2 - 36$

$36 = 4(x+1)^2$

$9 = (x+1)^2$

$\pm 3 = x + 1$ $x = -1 \pm 3$ $x = 2, -4$.

d) $j(x) = -2(x+3)^2 + 11$

e) $k(x) = -\frac{1}{3}\left(x - \frac{1}{2}\right)^2$

y-int → $y = -\frac{1}{3}\left(-\frac{1}{2}\right)^2 = -\frac{1}{12}$

x-int → $0 = -\frac{1}{3}(x - \frac{1}{2})^2$ $x = \frac{1}{2}$

f) $l(x) = x^2 + 5x$

g) $m(x) = -2x^2 - 4x - 6$

y-int → -6.

x-int → $0 = -2x^2 - 4x - 6$.

$0 = x^2 + 2x + 3$

h) $n(x) = \frac{1}{2}x^2 - 3x + \frac{5}{2}$

i) $p(x) = -3x^2 - 5x$

y-int → 0

x-int → $0 = -3x^2 - 5x$.

$0 = x(-3x - 5)$.

$x = 0, -5/3$.

j) $q(x) = -\frac{1}{2}x^2 - 4x - \frac{19}{3}$

k) $r(x) = -0.4x^2 + 2$

y-int → 2.

x-int → $0 = -0.4x^2 + 2$

$-2 = -0.4x^2$

$5 = x^2$

$x = \pm\sqrt{5}$.

l) $s(x) = 6x^2 - \sqrt{2}x - 2$

6. Graph each quadratic function. Find the vertex, axis of symmetry, x-intercept(s), and y-intercept of the graph.

a) $f(x) = -x^2 - 4x$

$f(x) = -(x^2 + 4x + 4 - 4)$

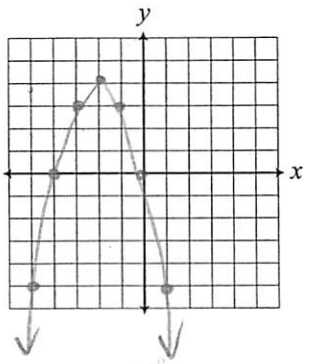
$f(x) = -(x+2)^2 + 4$

vertex $(-2, 4)$

axis of symmetry $x = -2$

x-intercept(s) $(-4, 0)(0, 0)$

y-intercept $(0, 0)$



b) $g(x) = -x^2 + 4x - 3$

$g(x) = -(x^2 - 4x + 4 - 4) - 3$

$g(x) = -(x-2)^2 + 4 - 3$

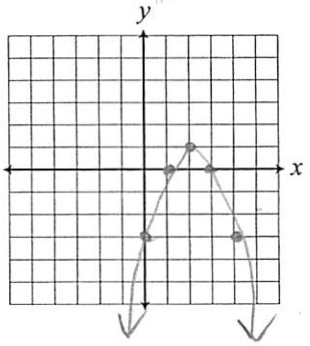
$g(x) = -(x-2)^2 + 1$

vertex $(2, 1)$

axis of symmetry $x = 2$

x-intercept(s) $(1, 0)(3, 0)$

y-intercept $(0, -3)$



c) $h(x) = x^2 - 4x + 5$

$h(x) = (x^2 - 4x + 4 - 4) + 5$

$h(x) = (x-2)^2 - 4 + 5$

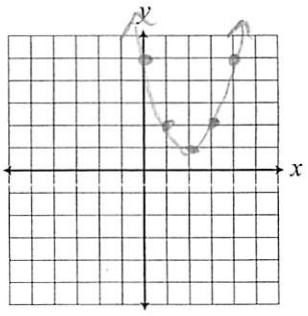
$h(x) = (x-2)^2 + 1$

vertex $(2, 1)$

axis of symmetry $x = 2$

x-intercept(s) none

y-intercept $(0, 5)$



d) $i(x) = -2x^2 - 4x + 3$

$h(x) = -2(x^2 + 2x + 1 - 1) + 3$

$h(x) = -2(x+1)^2 + 2 + 3$

$h(x) = -2(x+1)^2 + 5$

$0 = -2(x+1)^2 + 5$

$-5 = -2(x+1)^2$

$\frac{5}{2} = (x+1)^2$

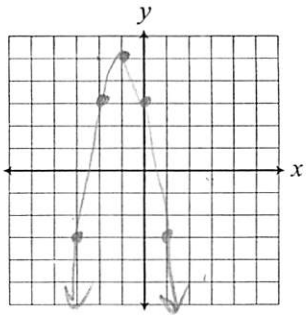
$\pm\sqrt{\frac{5}{2}} = x + 1$

vertex $(-1, 5)$

axis of symmetry $x = -1$

x-intercept(s) $\left(-1 \pm \sqrt{\frac{5}{2}}, 0\right)$

y-intercept $(0, 3)$



e) $j(x) = 3x - x^2$

$j(x) = -(x^2 - 3x + \frac{9}{4} - \frac{9}{4})$

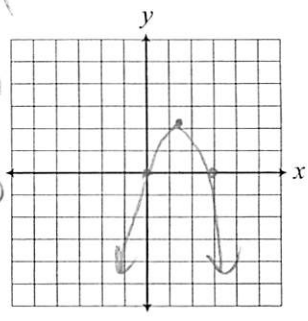
$j(x) = -(x - \frac{3}{2})^2 + \frac{9}{4}$

vertex $\left(\frac{3}{2}, \frac{9}{4}\right)$

axis of symmetry $x = \frac{3}{2}$

x-intercept(s) $(0, 0)(3, 0)$

y-intercept $(0, 0)$



6. f) $k(x) = -2x^2 - 8x - 6$

$k(x) = -2(x^2+4x+4-4) - 6$

$k(x) = -2(x+2)^2 + 8 - 6$

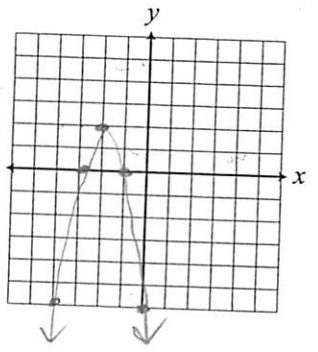
$k(x) = -2(x+2)^2 + 2$

vertex $(-2, 2)$

axis of symmetry $x = -2$

x-intercept(s) $(-3, 0)(-1, 0)$

y-intercept $(0, -6)$



g) $l(x) = -3x^2 + 6x + 3$

$l(x) = -3(x^2-2x+1-1) + 3$

$l(x) = -3(x-1)^2 + 3 + 3$

$l(x) = -3(x-1)^2 + 6$

$-6 = -3(x-1)^2$

$2 = (x-1)^2$

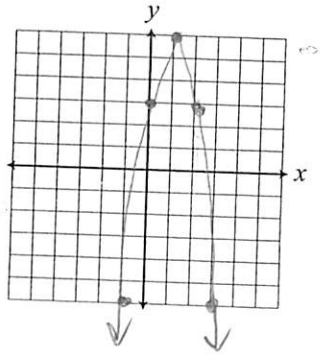
$\pm\sqrt{2} = x - 1$

vertex $(1, 6)$

axis of symmetry $x = 1$

x-intercept(s) $(1 \pm \sqrt{2}, 0)$

y-intercept $(0, 3)$



h) $m(x) = -\frac{1}{3}x^2 - 2x + 1$

$m(x) = -\frac{1}{3}(x^2+6x+9-9) + 1$

$m(x) = -\frac{1}{3}(x+3)^2 + 3 + 1$

$m(x) = -\frac{1}{3}(x+3)^2 + 4$

$-4 = -\frac{1}{3}(x+3)^2$

$12 = (x+3)^2$

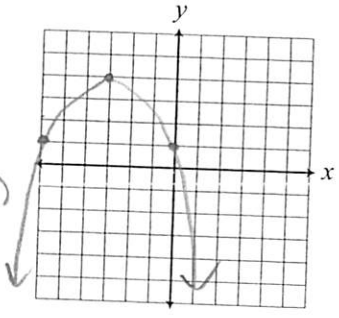
$\pm 2\sqrt{3} = x + 3$

vertex $(-3, 4)$

axis of symmetry $x = -3$

x-intercept(s) $(-3 \pm 2\sqrt{3}, 0)$

y-intercept $(0, 1)$



i) $n(x) = |x^2 - 4|$

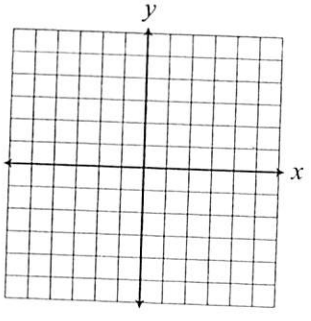
skip.

vertex _____

axis of symmetry _____

x-intercept(s) _____

y-intercept _____



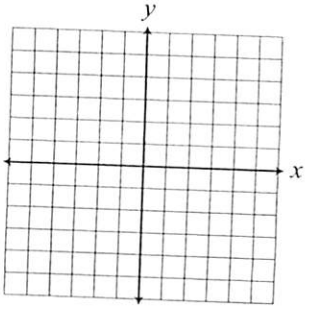
j) $p(x) = |-x^2 + 2x + 3|$

vertex _____

axis of symmetry _____

x-intercept(s) _____

y-intercept _____



7. Find the vertex of $f(x) = x^2 + kx + 4$.

$f(x) = (x^2 + kx + \frac{k^2}{4} - \frac{k^2}{4}) + 4$

$= (x + \frac{k}{2})^2 - \frac{k^2}{4} + 4$

$V(-\frac{k}{2}, \frac{-k^2}{4} + 4)$

8. Find the vertex of $g(x) = 2x^2 + kx + k^2$.

9. Find the vertex of $h(x) = 2x^2 + ax + b^2$.

$h(x) = 2(x^2 + \frac{a}{2}x + \frac{a^2}{16} - \frac{a^2}{16}) + b^2$

$h(x) = 2(x + \frac{a}{4})^2 - \frac{a^2}{8} + b^2$

$V(\frac{-a}{4}, \frac{-a^2}{8} + b^2)$

10. Find the vertex of $i(x) = px^2 - 3x + p$.

11. Find the vertex of $j(x) = kx(8 - x)$.

$j(x) = 8kx - kx^2$

$j(x) = -k(x^2 - 8x + 16 - 16)$

$j(x) = -k(x-4)^2 + 16k$

$V(4, 16k)$

12. Find a such that $f(x) = ax^2 + 4x - 4$ has a maximum value at $x = 6$.

13. Find b so that the function $f(x) = 2x^2 + bx - 3$ has a minimum value of -5.

$f(x) = 2(x^2 + \frac{b}{2}x + \frac{b^2}{16} - \frac{b^2}{16}) - 3$

$f(x) = 2(x + \frac{b}{4})^2 - \frac{b^2}{8} - 3$

$-\frac{b^2}{8} - 3 = -5$

$-\frac{b^2}{8} = -2 \quad -b^2 = -16$

$b^2 = 16 \quad b = \pm 4$

14. Find c so that the function $f(x) = 0.1x^2 + 7x + c$ has a minimum value of -120.5.

5.6 Exercise Set

1. The Acme automobile company has found that the revenue from sales of cars is a function of the unit price p that it charges. If the revenue, R, is $R = -\frac{1}{2}p^2 + 2000p$, what unit price, p, should be charged to maximize revenue? What is the maximum revenue?

$R = -\frac{1}{2}(p^2 - 4000p + 4{,}000{,}000 - 4{,}000{,}000)$

$+4{,}000{,}000 \rightarrow -2000$

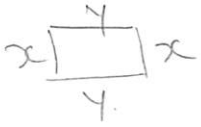
$R = -\frac{1}{2}(p-2000)^2 + 2{,}000{,}000$

price → \$2000

max → \$2,000,000

2. A cattle ranch with 6000 metres of fencing wants to enclose a rectangular feedlot that borders on a river. If the cattle will not go in the river, what is the largest area that can be enclosed?

3. Find the rectangle of maximum area that can be constructed with a perimeter of 36 cm.



$A = xy$

$36 = 2x + 2y$

$36 - 2x = 2y$

$18 - x = y$

$A = x(18-x) \quad A = 18x - x^2$

$A = -(x^2 - 18x + 81 - 81) \rightarrow A = -(x-9)^2 + 81$

4. What is the minimum product of two numbers that differ by 8? What are the numbers?

$y = 9$cm when $x = 9$cm → max area

5. A cannon shell is fired from a cliff 100 m above the water. The height, h, of the cannon above the water is given by $h = -0.005x^2 + x + 100$, where x is the horizontal distance of the cannon from the base of the cliff in metres.

a) How far from the base of the cliff is the height of the cannon shell at a maximum height?

b) Find the maximum height of the cannon shell.

c) How far from the base of the cliff will the cannon shell land in the water?

$h = -0.005(x^2 - 200x + 10000 - 10000) + 100$

$-10000 \rightarrow -100$

$h = -0.005(x-100)^2 + 50 + 100$

$h = -0.005(x-100)^2 + 150$

a) 100 m →

b) 150 m →

c) when $h = 0$

$-150 = -0.005(x-100)^2$

$30000 = (x-100)^2$

$\pm\sqrt{30000} = x - 100$

$x = 100 \pm \sqrt{30000}$

6. The school play charges \$10 for admission, and on average 80 people attend each show. For each \$1 increase, attendance drops by 5 people. What price should the school charge to maximize revenue?

= 273 m

7. A company can sell x stereos at a price of $\$(500 - x)$. How many stereos must be sold to maximize income? Find the maximum income.

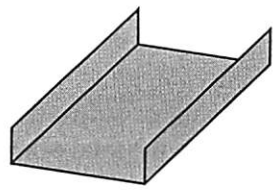
$R = (500 - x)x = -x^2 + 500x$

$R = -(x^2 - 500x + 62500 - 62500)$

$R = -(x - 250)^2 + 62,500$

250 stereo — max income.

8. A 20 cm wide sheet of metal is bent into a rectangular trough. How high must the trough be to maximize the trough's volume?



9. The sum of two integers is 10, and the sum of their squares is a minimum. Find the integers.

#1) x

#2) $10 - x$

$x^2 + (10 - x)^2$

$S(x) = x^2 + 100 - 20x + x^2 = 2x^2 - 20x + 100.$

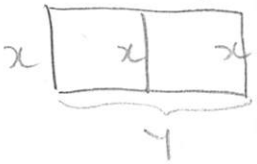
$S(x) = 2(x^2 - 10x + 25 - 25) + 100$

$= 2(x - 5)^2 - 50 + 100$

$= 2(x - 5)^2 + 50 \quad x = 5 \therefore$ both # are 5.

10. A parabolic arch has a span of 50 m and a maximum height of 10 m. What is the height of the arch at a point 10 m from the centre?

11. A rancher has 1200 m of fencing to enclose two adjacent rectangular corrals. What dimensions will produce a maximum enclosed area if the common sides are of equal length?



$1200 = 3x + 2y$

$A = xy$

$y = \frac{1200}{2} - \frac{3x}{2}$

$y = 600 - \frac{3x}{2}$

$A = x(600 - \frac{3x}{2})$

$A = -\frac{3x^2}{2} + 600x = -\frac{3}{2}(x^2 - 400x + 40000 - 40000)$

$A = -\frac{3}{2}(x - 200)^2 + 60\,000 \quad x = 200\text{ m}$

$y = 300\text{ m}$

12. Semiahmoo Fish and Game Club charges its members \$200 per year. For each increase over 60 members, the membership cost is decreased by \$2. What number of members would produce the maximum revenue for the club?

13. When priced at \$10, one type of software has annual sales of 300 units. For each dollar the software is increased in price, the store expects to lose the sale of 10 units of software. Find the price that will maximize the total revenue.

where x = # of increases

$R = (10+x)(300-10x)$

$R = 3000 - 100x + 300x - 10x^2$

$R = -10(x^2 - 20x + \underline{100} - \underline{100}) + 3000$

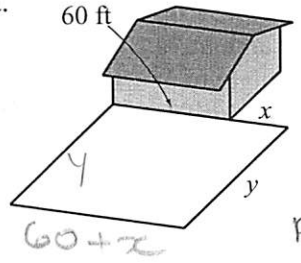
$R = -10(x-10)^2 + 4000$

$x = 10$ increases

Price $= 10 + 10 = \$20$.

14. Ship A is 50 nautical miles west of ship B. Ship A is heading east at 10 knots and ship B is heading south at 5 knots. Find the minimum distance between the ships, and at what time it occurred.

15. An equestrian club wants to construct a corral next to the barn, using the side of the barn. They have 300 ft of fencing. Find the dimensions that give the maximum area.



$A = y(60+x)$

$2y + 60 + x + x = 300$

$2y + 2x = 240$

$y = 120 - x$.

$A = (120-x)(60+x)$

$A = 7200 + 120x - 60x - x^2$

$A = -(x^2 - 60x + \underline{900} - \underline{900}) + 7200$

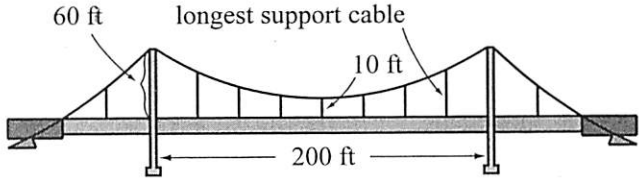
$A = -(x-30)^2 + 900 + 7200$

$A = -(x-30)^2 + 8100$

when $x = 30$m.

90m × 90m.

16. The cable supporting a suspension bridge is parabolic in shape. The support cables are vertical cables 25 ft apart. If the shortest support cable is 10 ft long, how long is the longest support cable?



17. A Norman window is a rectangle with a semi-circle on top. If the perimeter of the window is 24 ft, what dimensions will maximize the area of the window?

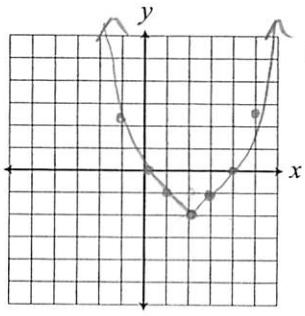
18. A piece of string 36 cm in length is cut into two pieces. One piece forms a square and the other piece a circle. How should the string be cut so the sum of the areas are a minimum?

5.7 Chapter Review

Section 5.1

1. Graph and state the vertex, axis of symmetry, x-intercept(s), y-intercept, domain, range, and maximum/minimum value of the following quadratic functions.

a) $f(x) = \frac{1}{2}x^2 - 2x$



$= \frac{1}{2}(x^2 - 4x + 4 - 4)$

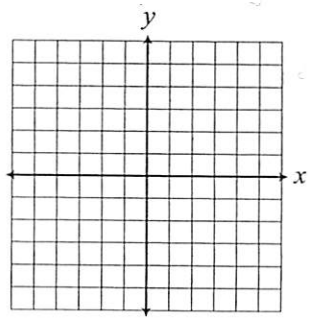
$\frac{1}{2}(x-2)^2 - 2$

$V(2,-2)$

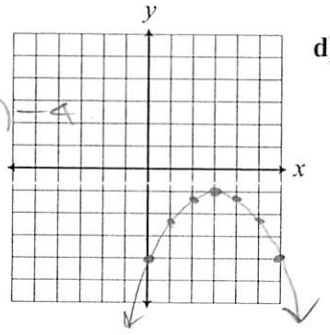
aos: $x=2$ x-int: $(0,0)$ $(4,0)$

y-int: $(0,0)$ D: $\mathbb{R}$ R: $y \geq -2$ min -2

b) $g(x) = -\frac{1}{2}x^2 + 3$



c) $y = -\frac{1}{3}x^2 + 2x - 4$



$= -\frac{1}{3}(x^2 - 6x + 9 - 9) - 4$

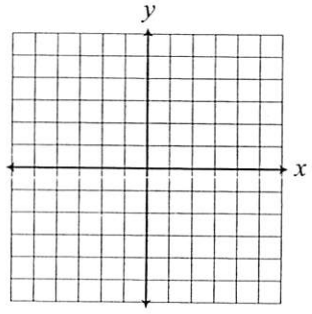
$= -\frac{1}{3}(x-3)^2 + 3 - 4$

$= -\frac{1}{3}(x-3)^2 - 1$

$V(3,-1)$ D: $\mathbb{R}$ R: $y \leq -1$ max -1

aos: $x=3$ x-int: none y-int: $(0,-4)$

d) $y = \frac{1}{2}x^2 - x - 4$



2. Find another point on the quadratic function, if the vertex and a point on the parabola are given.

a) vertex $(-2, 4)$, point $(0, 0)$ $(-4, 0)$

b) vertex $(0, -4)$, point $(-3, 0)$ _____

c) vertex $(3, 1)$, point $(-3, 4)$ $(9, 4)$

d) vertex $(-5, 3)$, point $(2, -4)$ _____

Section 5.2

3. For the graph of each function, state the vertex, axis of symmetry, x-intercept(s), y-intercept, domain, range, and maximum/minimum value.

a) $y = 2(x-1)^2 + 3$ V(1, 3)

AOS: $x=1$ D: $\mathbb{R}$ R: $y \geq 3$. min 3.

y-int: $y = 2(-1)^2 + 3 = 5$ (0, 5)

x-int: $-3 = 2(x-1)^2$ $-\frac{3}{2} = (x-1)^2$ ∴ none.

b) $y = -(x+2)^2 + 1$

c) $y + 4 = (x+1)^2$ -4

V(−1, −4) AOS: $x=-1$ D: $\mathbb{R}$ R: $y \geq -4$ min −4

y-int: $y = (1)^2 - 4 = -3$

x-int: $4 = (x+1)^2$ $\pm 2 = x+1$ $x = -1 \pm 2$ $x = 1, -3$

d) $y = -2(x+3)^2 + 8$

4. Write an equation of a parabola that has the shape $f(x) = \pm\frac{1}{2}x^2$ with the given vertex. (1, 0) (−3, 0)

a) minimum $(3, -2)$

$y = \frac{1}{2}(x-3)^2 - 2$

b) maximum $(1, 4)$

c) minimum $(-1, 5)$

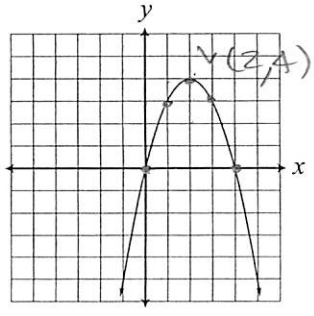
$y = \frac{1}{2}(x+1)^2 + 5$

d) maximum $(-3, -1)$

Section 5.3

5. Determine the equation of the parabola in the form $f(x) = a(x-h)^2 + k$.

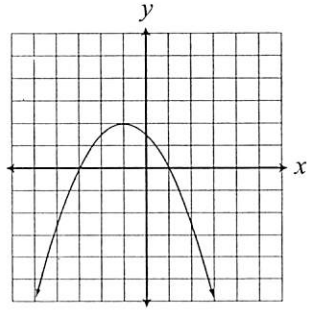
a)



$a = -1$

$y = -(x-2)^2 + 4$.

b)



6. Find the equation of a quadratic function whose graph satisfies the given conditions.

a) vertex: $(3,-4)$, x-intercept: 2 → $(2,0)$

$0 = a(2-3)^2 - 4$

$4 = a(-1)^2$

$4 = a$ $\quad y = 4(x-3)^2 - 4$

b) vertex: $(-2,1)$, x-intercept: -3

c) vertex: $(-1,-4)$, point: $(2,-1)$ $\quad x \quad y$

$-1 = a(2+1)^2 - 4$

$-1 = a(3)^2 - 4$

$3 = 9a$

$1/3 = a$ $\quad y = \frac{1}{3}(x+1)^2 - 4$

d) $(-2,0),(4,0),(0,3)$

Section 5.4

7. Find the vertex, x-intercept(s) (if possible), and y-intercepts for each quadratic function.

a) $f(x) = -2x^2 - 6x - 4$

$f(x) = -2(x^2 + 3x + \frac{9}{4} - \frac{9}{4}) - 4$

$= -2(x+\frac{3}{2})^2 + \frac{9}{2} - 4$

$f(x) = -2(x+\frac{3}{2})^2 + \frac{1}{2}$

$V(-3/2, 1/2)$

b) $g(x) = \frac{1}{2}x^2 - \frac{5}{2}x + \frac{25}{8}$

y-int: $y = -2(\frac{3}{2})^2 + \frac{1}{2}$

$= -4$ $(0,-4)$

x-int: $-\frac{1}{2} = -2(x+\frac{3}{2})^2$

$\pm\sqrt{\frac{1}{4}} = x + \frac{3}{2}$ $\quad x = -\frac{3}{2} \pm \frac{1}{2} = -1, -2$ $(-1,0)$ $(-2,0)$

8. Graph each quadratic function. Find the vertex, axis of symmetry, x-intercept(s), and y-intercept of the graph.

a) $f(x) = -\frac{1}{3}x^2 + x$

$f(x) = -\frac{1}{3}(x^2 - 3x + \frac{9}{4} - \frac{9}{4})$ $\quad \frac{-3}{2}$

$f(x) = -\frac{1}{3}(x-3/2)^2 + \frac{3}{4}$

y-int: $(0,0)$

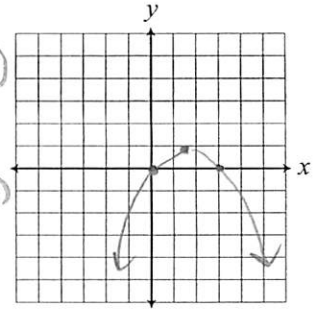
x-int: $-\frac{3}{4} = -\frac{1}{3}(x-3/2)^2$ $\quad \pm\sqrt{\frac{9}{4}} = x - 3/2$ $\quad x = \frac{3}{2} \pm \frac{3}{2} = 3, 0$ $(3,0)(0,0)$

vertex $(3/2, 3/4)$

axis of symmetry $x = 3/2$

x-intercept(s) $(3,0)(0,0)$

y-intercept $(0,0)$



b) $g(x) = \frac{1}{2}x^2 - x - 4$

$g(x) = \frac{1}{2}(x^2 - 2x + 1 - 1) - 4$

$= \frac{1}{2}(x-1)^2 - \frac{1}{2} - 4$

$= \frac{1}{2}(x-1)^2 - \frac{9}{2}$

y-int: $\frac{1}{2}(-1)^2 - \frac{9}{2} = -4$

x-int: $\frac{9}{2} = \frac{1}{2}(x-1)^2$ $\quad 9 = (x-1)^2$ $\quad x = 1 \pm 3$

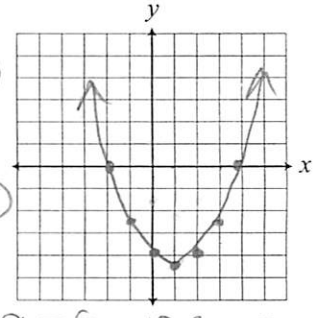
$\pm 3 = x - 1$ $\quad x = -2, 4$.

vertex $(1, -9/2)$

axis of symmetry $x = 1$

x-intercept(s) $(-2,0)(4,0)$

y-intercept $(0, -9/4)$



Section 5.5

9. Find the vertex of each quadratic function.

a) $f(x) = 2x^2 - 12x + 7$

b) $g(x) = 3x^2 + 6x + 2$

c) $h(x) = -2x^2 + 12x - 20$

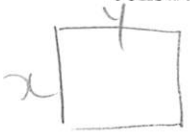
d) $i(x) = -2 + 2x - \frac{1}{2}x^2$

10. If $(8, 0)$ is on $y = bx + x^2$, what is the least value of the function?

11. If $(-6, 0)$ is on $y = bx - x^2$, what is the greatest value of the function?

Section 5.6

12. Find the rectangle of maximum area that can be constructed with a perimeter of 44 cm.



$P = 44 = 2x + 2y$

$A = xy$

$24 = 44 - 2x$

$y = 22 - x$

$A = x(22 - x) = -x^2 + 22x$

$A = -(x^2 - 22x + 121 - 121)$

-11

$A = -(x - 11)^2 + 121$

when $x = 11$ cm

$y = 11$ cm

13. A bus touring company charges \$10 per passenger, and carries an average of 300 passengers per day. The company estimates it will lose 15 passengers for each increase of \$1 in the fare. What is the most profitable fare to charge?